A Fair Payment Scheme for Virtuous Community Energy Usage

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Abstract

In this work, we face a payment estimation problem that involves a community of users and an energy distributor (or producer). Our aim is to compute payments for every user in the community according to the single user's consumption, the community's consumption and the available energy. The proposed scheme influences the community in consuming in a virtuous way. In order to reach this goal, our payment function distributes incentives if the consumption is lower than the produced energy and penalties when the consumption exceeds the resources threshold. Our model satisfies efficiency and fairness properties both from the community (efficiency as an economic equilibrium among sellers and buyers) and the single user (fairness as an economic measure of energy good-behaving) points of view. By computing community-dependent energy bills, our model stimulates a virtuous users' behaviour so that it approaches the production threshold as close as possible. We also provide a simulation based on real data referring to a dataset of buildings in California State, thus, showing several possible shapes of our payment scheme.

1 Introduction

In a renewable world, energy production changes day by day thus we have to quickly adequate energy requests. For instance, the European Commission (EU) finances projects in which users are stimulated to behave in an energy-aware manner according to energy saving targets (20% cut in greenhouse emissions, Targets 2020 (Bohringer, Rutherford, and Tol 2009)). Over the years several approaches has been deployed by researchers to address this problem, in our case, we face a payment estimation problem that involves a community of users and a set of divisible resources (in our case, electrical energy). Thus, we provide a payment function for users, that takes into account not only the single user's consumption (as it usually happens) but also other users' consumption and the total produced energy.

So, our main objective is to select proper payments relying on community consumption and available energy. Thus, we define payments assigning bonus or penalties when the energy is consumed efficiently (that this, to pay more than the consumption if the consumption exceeds the production, and vice-versa, to pay less than the consumption when production exceeds consumption).

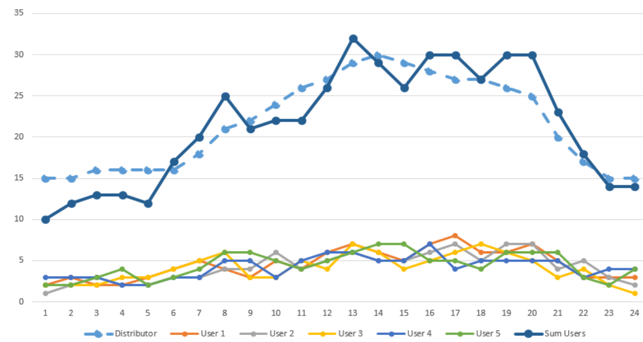

Figure 1: Comparison between energy functions: users' single trends (coloured lines on the bottom), the aggregated trend of all agents (blue continuous line) and the produced energy function (light blue dashed line).

The presented payment scheme satisfies efficiency and fairness properties at two different levels: community and single user. In fact, for instance regarding efficiency, we guarantee that at the community level the amount of money exchanged with the distributor will be very similar as having a fixed energy price. On the other hand, efficiency at single user level means that at every time the amount paid to the distributor is shared entirely among users.

Fig. 1 describes a possible case for a one-day time period with trends for the energy functions. The dashed line represents the distributor's available energy and the continuous line is the energy consumed by the entire community. The lines on the bottom represent the single consumption of every user (five users in this case). Considering this figure, our objective is to increase the energy price where the continuous line is over the dashed line and to decrease it in the opposite case. In this way, the energy distributor can indirectly control the loads in the user premises. This approach is known in the literature as a demand response (DR) program (Albadi and El-Saadany 2007). In particular, we refer to one of the categories of DR mechanisms called "price- and incentive-based DR methods"(Albadi and El-Saadany 2007). It deals with the definition of incentive pricing scheme to influence users' behaviour. In fact, a generic user may wish to change his energy behaviour for saving money. Moreover, at community level, fairness means that

if the community does not consume, it will not pay any amount, analogously at single user level if a consumer does not consume energy he will not pay any amount.
We want to underline the importance of taking into account the community instead of only single users. In fact, in real world, it can happen that a certain amount of energy is bought not only by a single user but by a community, that is an ensemble of consumers (i.e.: an interconnected energy smart grid). For this reason, we face two related problems: how much a community should pay for its consumption and how to divide this global payment among users in the community.
The paper is divided in the following way: in Section 2, we give a brief background on cooperative game theory. Section 3 describes our model in details. In Section 4, we show graphically possible payment functions. We present related works in Section 5. Finally, in Section 6 we conclude the paper with some remarks and future works.

2 Background

2.1 Cost-Sharing Game

The notion of a cost-sharing game (or cooperative game) (Nisan et al. 2007; Narahari 2014) was first proposed by Von Neumann and Morgenstern (Von Neumann and Morgenstern 1944). This approach considers combinatorial aspects of coalitions as a central point. Thus, it differs from the other part of game theory that deals with a competitive approach where players' strategies and equilibrium aspects are faced.
In details, suppose to have a set $G = \{1, \ldots, n\}$ of n agents seeking to cooperate in order to generate value and a coalition $M \subseteq G$. The composition of coalition M influences consistently the generated value. We can compute this value through a valuation function v defined on the power set of G. The setting (G, v) constitutes a classical cooperative game with non-transferable utilities (abbreviated an NTU game). A special case, called a cooperative game with transferable utilities (abbreviated as a TU game), is when the value generated by a coalition can be arbitrarily divided among the agents in G. In other words, a TU game is defined by specifying a function $v : 2^G \to \mathbb{R}$, assuming $v(\emptyset) = 0$. Usually function v computes the value generated by a coalition,if we are evaluating costs we formalize a cost-sharing game, based on cost function $c : 2^G \to \mathbb{R}$ where $C = \{c : 2^G \to \mathbb{R}\}$. A general cooperative problem can be studied in both the TU and the NTU models. The TU model applies to settings where, for example, a service provider incurs some (monetary) cost c in building a network that connects a set M of customers to the Internet, and needs to divide this cost among customers in M. Fixing a set $G = \{1, \ldots, n\}$ of n agents and a cost function c (or generally a value function v), we can define several properties:

Definition 1 (Efficiency (Nisan et al. 2007)) *A partition is a function that assigns to each cost function c a vector $x(c) \in \mathbb{R}^n$ of nonnegative numbers. If all the cost is shared by the users, $\sum_{i \in G} x_i = c(G)$ we say that the cost division $x(c)$ is efficient.*

Definition 2 (Dummy (Nisan et al. 2007)) *The cost division $x(c)$ satisfies the dummy properties if for every agent who does not add to the cost should not be charged anything. More precisely, if for every set $S \subset G \setminus \{i\}$, $c(S) = c(S \cup \{i\})$, then $x_i(c) = 0$.*

Definition 3 (Anonymity (Nisan et al. 2007)) *The cost division $x(c)$ satisfies the anonymity properties if changing the names of the agents does not change their cost shares. Formally, x satisfies anonymity if for every permutation π of G and every cost function c, $x_{\pi_i}(\pi(c)) = x_i(c)$ for every $i \in G$.*

Definition 4 (Additivity (Nisan et al. 2007)) *The cost division $x(c)$ satisfies the additivity properties if for every two cost functions c_1 and c_2, $x(c_1 + c_2) = x(c_1) + x(c_2)$, where $c_1 + c_2$ is the cost function defined by $(c_1 + c_2)(M) = c_1(M) + c_2(M)$.*

Shapley Value In this section, we study a solution concept called the Shapley value (Shapley 1953) that assigns a single cost allocation to any given cost-sharing game. The solution concept satisfies fairness (dummy, anonymity, additivity) and efficiency properties which are concepts extracted from economic theory.
Assume to have a classical cost-sharing game (G, c). The simplest solution to share cost is to select an order (for instance, "$1, 2, ..., n$") and calculate the marginal cost adding a player at every step. Thus, user 1 will get a cost of $c(\{1\})$, then user 2 of $c(\{1, 2\}) - c(\{1\})$ and so on. This is an efficient method, but it is clear that it do not satisfy a fairness property. In fact, it is not anonymous because the amount charged strictly depends on the ordering of the agents, i.e. on player's position in the ordering. The Shapley value overcomes this approach by considering all the possible orders computing for each them the marginal cost. Formally, suppose to have for every $i \in G$ a set S not containing user i, $S \subseteq G \setminus \{i\}$. First of all, we compute the probability that the set of agents that come before i in a random ordering that is precisely S is $s!(n-1-s)!/n!$. This is probability is deployed by the Shapley value in the following formula, for each agent $i \in G$:

$$\phi_i(c) = \sum_{s=0}^{n-1} \frac{s!(n-1-s)!}{n!} \cdot \sum_{\substack{S \subseteq G \setminus \{i\} \\ |S| = s}} (c(S \cup \{i\}) - c(S)) \tag{1}$$

where $\phi_i(c) : C \to \mathbb{R}$ indicates the cost share of $i \in G$ in the cost-sharing game (G, c). It calculates and sums for every index s the probability that the set of agents that come before i in a random ordering multiplied by the marginal contribution considering all the possible combinations of coalitions of size s. It is possible to prove that the Shapley value satisfies all the properties described in Def. 1,2,3,4 (Nisan et al. 2007).

Theorem 1 *The Shapley value is the unique value satisfying efficiency, anonymity, dummy, and additivity.*

Proof 1 *The proof can be found in (Nisan et al. 2007).*

Example 1 (Airport Problem) *The airport problem (Littlechild and Owen 1973) is a type of fair cost division problem in which it is decided how to distribute the cost of an airport runway among different players who need runways of different lengths. An airport needs to build a runway for 4 different aircraft types. The building cost associated with each aircraft is 8$, 11$, 13$, 18$ for aircraft A, B, C, D. Considering these costs the Shapley value for the aircraft A will be:*

$\phi_A(c) = \frac{3!}{4!}\cdot 8 + \frac{2!}{4!}\cdot[(11-11)+(13-13)+(18-18)] + \frac{2!}{4!}\cdot[(13-13)+(18-18)+(18-18)] + \frac{3!}{4!}\cdot(18-18) = \frac{1}{4}\cdot 8 = 2$

In the same way calculating other Shapley values with Eq. 1, we would come up with the following cost table:

Aircraft	***Building Cost***	***Shapley value***
A	*8*	*2*
B	*11*	*3*
C	*13*	*4*
D	*18*	*9*
Total		***18***

Table 1: Cost function and Shapley values of the Airport Problem.

In this table, we can see that the total cost (18), coinciding with the building cost of the longest runway is divided between the aircraft type according to their influence in the total cost. For instance, the type C contributes with 4 over 18 according to its building cost of 13.

3 Model Description

In the definition of the payment scheme, we start from the calculation of the aggregated consumption $\theta(t) = \sum_{i\in G}\theta_i(t)$, where $\theta_i : T \to \mathbb{R}$ is the consumption profile for user $i \in G$ and G is the collection of users (community). We suppose that the money charged to users is directly correlated to the consumption, thus, θ represents also the distributor's income for every $t \in T$.

In order to compute incentives or penalties, we define an energy availability function $p : T \to \mathbb{R}$, determining the produced energy value. Function p allow us to define the energy cost function $\psi_p : \mathbb{R} \to \mathbb{R}$. The idea behind the design of the function ψ_p is to give an incentive in terms of money at time t in which the overall consumption θ is under function p and to get the same amount of money back by increasing the energy price at time t in which θ exceeds function p. Moreover, we design a function that is increasing according to consumption, i.e.: the more amount of energy users consume, the bigger will be the cost. We also assume that the total available energy is exactly the aggregated consumed energy. Formally, we assume that the amount of energy consumed is equal to the energy provided:

$$\int_T \theta(t)dt = \int_T p(t)dt \quad (2)$$

We can now introduce fairness and efficiency properties, one of them is *community dummy*: a community dummy property ensures that if the community does not consume energy, it will not be charged.

Definition 5 (Community Dummy) *A particular payment scheme ψ_p satisfies the community dummy property if*

$$\psi_p(\theta(t)) = 0 \quad \text{if } \theta(t) = 0. \quad (3)$$

Furthermore, as one of our aim is to design a fair energy payment scheme, we must satisfy the constraint that if the consumption is equal to function p the community have to pay as having a fixed energy price.

Definition 6 (Community Fairness) *A particular payment scheme ψ_p satisfies the community fairness property if it satisfies:*

$$\psi_p(\theta(t)) = \theta(t) \quad \text{if } \theta(t) = p(t). \quad (4)$$

As stated before, a scheme can maintain the distributor's income as in a fixed energy price case. For this reason, assuming an energy cost equal to one, we formalize this constraint in the following way:

Definition 7 (Community Efficiency) *A particular payment scheme ψ_p satisfies the community efficiency property if the quantity of money exchange does not change:*

$$\int_T \theta(t)dt = \int_T \psi_p(\theta(t))dt. \quad (5)$$

Eq. 5 states that the distributor will receive an amount equal to standard fixed price. In other words, distributor will not reduce his money income. Sometimes it is necessary to relax the above presented constraint, thus, we state that a payment scheme could not exactly cover the distributor income but its bounded below by an ϵ value.

Definition 8 (ϵ-Community Efficiency) *A particular payment scheme ψ satisfies the community efficiency property if the quantity of money exchange is bounded below by an ϵ value:*

$$\int_T \psi_p(\theta(t))dt \geq (1-\epsilon)\int_T \theta(t)dt. \quad (6)$$

Now, we start defining our payment function ψ_p. First of all, we select a comprehensible function structure presented in the following equation:

$$\psi_p(\theta(t)) = \alpha(\theta(t), p(t))\chi_{\theta(t)\leq p(t)} + \beta(\theta(t), p(t))\chi_{\theta(t)>p(t)}, \quad (7)$$

where χ_λ is the characteristic function of elements of the set that satisfies the condition λ and $\alpha, \beta : \mathbb{R} \to \mathbb{R}$ are arbitrary non-negative functions.

Function ψ_p represents the amount of cost to be assigned to the community at every time t and $\alpha, \beta : \mathbb{R} \to \mathbb{R}$ are arbitrary non-negative functions selected to give penalties or incentives. Function α will give incentives according to the gap to available energy; in the same way, function β will charge the community according to its excess from production function p.

We propose a function ψ_p with selected functions $\alpha(\theta, p) = \theta(t)\mathrm{e}^{a(\theta(t)-p(t))}$ as incentive function, $\beta(\theta, p) = \theta(t)\mathrm{e}^{b(\theta(t)-p(t))}$ as penalty function. Thus, the resulting function is:

$$\psi_p(\theta(t)) = \theta(t)\mathrm{e}^{a(\theta(t)-p(t))}\chi_{\theta\leq p} + \theta(t)\mathrm{e}^{b(\theta(t)-p(t))}\chi_{\theta>p}. \quad (8)$$

Theorem 2 (Community Dummy) *The function ψ_p presented in Eq. 8 satisfies the dummy property Def. 5.*

Proof 2 *For every $t \in T$ such that $\theta(t) = 0$ we have:* $\psi_p(0) = 0\mathrm{e}^{a(0-p(t))}\chi_{\theta(t)\leq p(t)} = 0\mathrm{e}^{-ap(t)} = 0.$ □

Theorem 3 (Community Fairness) *The function ψ_p presented in Eq. 8 satisfies the fairness property Def. 6.*

Proof 3 *For every $t \in T$ such that $\theta(t) = p(t)$ we have:* $\psi_p(\theta(t)) = \theta(t)\mathrm{e}^{a(\theta(t)-p(t))}\chi_{\theta(t)\leq p(t)} = \theta(t)\mathrm{e}^{a0} = \theta(t).$ □

Theorem 4 (ϵ-Community Efficiency) *The function ψ_p presented in Eq. 8 satisfies the ϵ-Community efficiency property Def. 8 with $\epsilon = 1 - \mathrm{e}^{-\hat{a}\max_T p(t)}$.*

Proof 4 *We can prove that:*

$$\int_T \psi_p(\theta(t))dt \geq$$

$$\int_{\theta(t)\leq p(t)} \theta(t)\mathrm{e}^{a(\theta(t)-p(t))}dt + \int_{\theta(t)>p(t)} \theta(t)\mathrm{e}^{b(\theta(t)-p(t))}dt \geq$$

$$\mathrm{e}^{-a\max\limits_T p(t)} \int_{\theta(t)\leq p(t)} \theta(t)dt + \mathrm{e}^{-b\max\limits_T p(t)} \int_{\theta(t)>p(t)} \theta(t)dt \geq$$

$$\mathrm{e}^{-\hat{a}\max\limits_T p(t)} \int_T \psi_p(\theta(t))dt \implies$$

$$\int_T \psi_p(\theta(t))dt \geq (1-\epsilon)\int_T \theta(t)dt$$

where $(1-\epsilon) = \mathrm{e}^{-\hat{a}\max\limits_T p(t)}$ *and* $\hat{a} = \max\{a, b\}$. □

The proposed function allows us to have a compromise among penalizations and incentives, in the sense that if the community is a bit under or over the threshold it will have a small incentive or penalty. The more the consumption is far from the threshold, the more the community will have a incentive/penalty.

3.1 Fair Cost Division

Function ψ represents the payment function of the community with respect to the overall consumption θ. Thus, what remains to do is to divide this cost among users. This can be done by the application of the Shapley value for a fair cost division. We assume that $\theta_S(t) = \sum_{i\in S}\theta_i(t)$ is the aggregated consumption for a set of users $S \subseteq G$. First of all, we have to define the function $\psi(\theta_S(t))$, that is:

$$\begin{aligned}\psi_p(\theta_S(t)) =& \theta_S(t)\mathrm{e}^{a(\theta_S(t)-p(t))}\chi_{\theta_S(t)\leq p(t)} \\ &+ \theta_S(t)\mathrm{e}^{b(\theta_S(t)-p(t))}\chi_{\theta_S(t)>p(t)}\end{aligned} \quad (9)$$

At this point, recalling the Shapley value scheme, we can divide cost among users in two different ways. We can suppose an instantaneous billing of consumed energy or we can compute a bill for the whole interval T. The first option is more precise because it calculates the Shapley value for every $t \in T$, supposing to have an end of period T bill, the second one has a lower computational cost because we calculate the Shapley value once for the whole interval T. In the following sections, we are going to show in details the two approaches.

Option 1 The first possibility is to use function ψ as cost function c to compute the aggregated cost for every $S \subseteq G$ and every $t \in T$. Thus, in this case we define the cost function $c_1 : 2^n \times T \to \mathbb{R}$ in the following way:

$$c_1(S,t) = \psi(\theta_S(t)). \quad (10)$$

Considering that this valuation function will be the input for the Shapley value scheme, we can write:

$$\phi_i(c_1) = \sum_{s=0}^{n-1} \frac{s!(n-1-s)!}{n!} \sum_{\substack{S\subseteq G\setminus\{i\} \\ |S|=s}} \Big(c_1(S\cup\{i\}) - c_1(S)\Big) \quad (11)$$

Therefore, amount ϕ_i represents the money charged on user i for the energy consumed at time t. By calculating the integral of ϕ_i in the interval T, we can find the total amount that user i should pay for the period T. For instance, if we fix $t = t_1$, we can compute the cost of $\psi(\theta(t_1))$ that can be divided among users with the Shapley value scheme.

Option 2 As second possibility we can assume that the cost function is computed over the integral of the function ψ, thus, the cost function $c_2 : 2^n \to \mathbb{R}$ is defined in the following way:

$$c_2(S) = \int_T \psi(\theta_S(t))dt \quad (12)$$

Therefore, in this second case amount ϕ_i represents the money charged on user i for the energy consumed during the whole interval T.

Theorem 5 (Integral Equivalence) *The allocations computed with the cost functions c_1, c_2 returns the same aggregated amount of money exchanged over interval T for every user $i \in G$, formally:*

$$\int_T \phi_i(c_1(S,t))dt = \phi_i(c_2(S)) \quad (13)$$

Proof 5 *It results by easily integrating Eq. 11.* □

3.2 Properties

In conclusion, suppose to choose the Option 1 regarding the cost function, the value ϕ_i has several properties proved in the following lines.

Definition 9 (Properties) *An energy payment scheme has the following properties:*

1. ***Irrelevant Consumer**, if a consumer has a consumption $\theta_i(t) = 0$ on some $t \in T$, his payment will be $\phi_i = 0$ for that t.*
2. ***Parity Consumer**, if a community has a consumption $\theta(t) = p(t)$, every user $i \in G$ will pay exactly $\theta_i(t)$.*
3. *ϵ**-Community Efficiency**, the total amount of money paid by users is bounded below by an ϵ value as having a fixed energy price, see Eq. 6.*
4. ***Nonlinearity**, the incentives or penalties are not linear with respect to the consumed energy. It means that $\psi(\theta_i(t) + \theta_j(t)) \geq \psi(\theta_i(t)) + \psi(\theta_j(t))$ for every user $i, j \in G$.*

5. ***Virtuousness***, *for every $t \in T$ a user will pay according to the consumption of the whole community and to his influence in the community. For instance, if the community consumes more than the defined threshold, it will receive a penalty shared among users. If a user has a much lower consumption than others, he will pay a much lower portion than the others. It means that $\phi_i(c) > \phi_j(c)$ iff $\theta_i(t) > \theta_j(t)$ for every $i, j \in G$.*

Theorem 6 (Properties) *The allocation computed by ϕ_i for every $i \in G$ described in Eq. 11 satisfies the following properties: irrelevant consumer, balanced money exchange, nonlinearity, virtuousness.*

Proof 6 *:*

1. *Irrelevant Consumer, the first property is guaranteed by two different perspectives. First of all, function ψ_p satisfies the community dummy property, see Th. 2. From a single user point of view, the dummy property satisfied by the Shapley value formula states that if for every set $S_{-i} \subset G \setminus \{i\}$, $c(S_{-i}) = c(S_{-i} \cup \{i\})$, then $\phi_i(c) = 0$. The implication $c(\emptyset) = 0$ which is the only required condition to apply Shapley value formula, is guaranteed by Th. 2.* □
2. *Parity Consumer, also this property is guaranteed by two different perspectives. First of all, function ψ_p satisfies the community fairness property, see Th. 3. Then, the SV efficiency property ensures that: $v(G, t) = \psi(\theta(t)) = \sum_{i \in G} \phi_i$. Thus, the unique possible allocation returned by the SV is: $\phi_i(c) = \theta_i(t)$.* □
3. *ϵ-Community Efficiency, the construction of function ψ_p is based on the selection of parameters a, b. According to Th. 4, we ensure that the money exchanged is at least $(1 - \epsilon) \int_T \theta(t)dt$. Then, the SV efficiency property Th. 1 ensures that: $v(G, t) = \psi(\theta(t)) = \sum_{i \in G} \phi_i$.* □
4. *Nonlinearity, as we want that the payment does not depend linearly on the energy consumed, we selected a nonlinear function ψ_p. Suppose to have only two users i, j.*
   - *Case I, Taking into account $t \in T$ where $\theta_i(t), \theta_j(t) \leq p(t)$ and $\theta_i(t) + \theta_j(t) \leq p(t)$, we show that:*

     $$\psi(\theta_i(t) + \theta_j(t)) \geq \psi(\theta_i(t)) + \psi(\theta_j(t)) \rightarrow$$
     $$(\theta_i(t) + \theta_j(t))e^{a(\theta_i(t)+\theta_j(t)-p(t))} \geq$$
     $$\theta_i(t)e^{a(\theta_i(t)-p(t))} + \theta_j(t)e^{a(\theta_j(t)-p(t))}$$

     Thus, the inequality ≥ 0 holds.
   - *In Case II, where $\theta_i(t), \theta_j(t) \leq p(t)$, thus, $\theta_i(t) + \theta_j(t) > p(t)$ is exactly the same by substituting a with b.*
   - *In Case III, where $\theta_i(t), \theta_j(t) \leq p(t)$ and $\theta_i(t) + \theta_j(t) > p(t)$ the nonlinearity is satisfied if:*

     $$(\theta_i(t) + \theta_j(t))e^{b(\theta_i(t)+\theta_j(t)-p(t))} \geq$$
     $$\theta_i(t)e^{a(\theta_i(t)-p(t))} + \theta_j(t)e^{a(\theta_j(t)-p(t))} \iff$$
     $$\theta_i(t)\left(e^{b(\theta_i(t)+\theta_j(t)-p(t))} - e^{a(\theta_i(t)-p(t))}\right) \geq$$
     $$\theta_j(t)\left(e^{b(\theta_i(t)+\theta_j(t)-p(t))} - e^{a(\theta_j(t)-p(t))}\right)$$

     the inequality holds because $e^{a(\theta_i(t)-p(t))} \leq 1 \leq e^{b(\theta_i(t)+\theta_j(t)-p(t))}$.
   - *In Case IV, it is $\theta_i(t) \leq p(t)$ and $\theta_j(t) > p(t)$ (that implies $\theta_i(t) + \theta_j(t) > p(t)$), we have:*

     $$(\theta_i(t) + \theta_j(t))e^{b(\theta_i(t)+\theta_j(t)-p(t))} \geq$$
     $$\theta_i(t)e^{a(\theta_i(t)-p(t))} + \theta_j(t)e^{b(\theta_j(t)-p(t))}$$

     here the inequality holds because $\theta_i(t)e^{a(\theta_i(t)-p(t))} \leq \theta_i(t) \leq \theta_i(t)e^{b(\theta_i(t)+\theta_j(t)-p(t))}$ and $\theta_j(t)e^{b(\theta_j(t)-p(t))} \leq \theta_j(t))e^{b(\theta_i(t)+\theta_j(t)-p(t))}$. □
5. *Virtuousness, suppose the case $\theta(t) < p(t)$, the community will receive an incentive in fact, we have $\psi(\theta(t)) = \theta(t)e^{a(\theta(t)-p(t))} \leq \theta(t)$. Thanks to application of Shapley value based on the marginal contribution $c(S \cup \{i\}) - c(S)$, our scheme ensures that $\phi_i(c) > \phi_j(c)$ iff $\theta_i(t) > \theta_j(t)$ for every $i, j \in G$. It means that a virtuous user, having a lower consumption, will get a greater incentive than a user having a greater consumption.* □

4 Experimental Results

In order to show the capabilities of the presented payment scheme, we tested our model in a real case based on a set of hourly data of the California State. Consumption data are extracted from a web source (OpenEI 2017 accessed July 31 2017) and the production profile coming from renewable energy sources is extracted from data provided by the California Independent System Operator (CISO 2017 accessed July 31 2017). We take into account a time period of 48 hours to evaluate also a daily seasonality. We randomly select 10 users' consumption profile and the production profile for the same period. It is crucial to note that we are interested in the shape of the production curve not in the actual amount of available energy. In fact, in this example, we want to provide payment functions to stimulate users in behaving as close as possible to the provided renewable production profile. In Fig. 2, we show our function ψ_p, which gives a consistent penalization in time slots in which consumption is much higher than the objective function. We draw several possible functions ψ according to how much we want to incentive/penalize the community by tuning parameters a and b. In addition, Fig. 3 shows the Shapley values for every user in the interval. The presented simple implementation has the aim to show graphically the potentiality of our payment scheme. In fact, thanks to the tuning of one parameter (for instance a calculating b according to Th. 4) we are able to propose an adaptive payment scheme.

5 Related Work

From the state of art, an exploratory study on the energy pricing information is (Ameren 2012). It is based on the statistical analysis, the authors argued that the current price of electricity depends on the prices of yesterday, the day before yesterday, and the same day of the past week. The paper developed a model based on the correlation of these three prices and three different coefficients to control the weight of these prices. This work represents an interesting

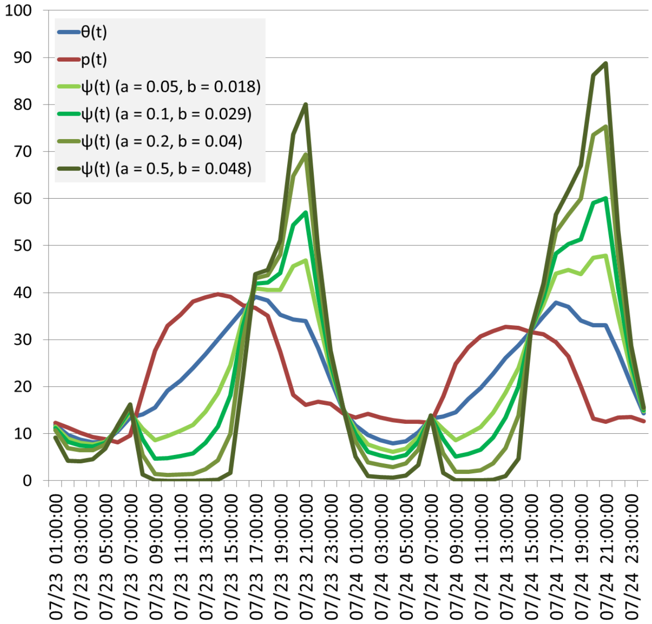

Figure 2: Several possible functions ψ (green lines) computed for community's consumption (blue line) and renewable production function (red line).

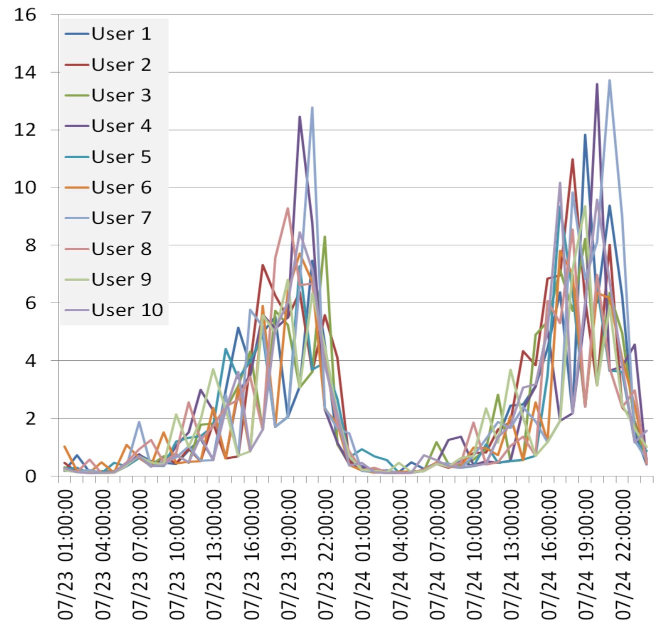

Figure 3: Single-user payment functions computed by the Shapley value formula considering function ψ_p with parameter $a = 0.1$ and $b = 0.029$.

predictive method for energy price but it does not deal with users' behavioural change. In (Rad and Leon-Garcia 2010), researchers tackle the DR problem by proposing an optimal and automatic residential energy consumption scheduling framework which attempts to achieve the desired trade-off between minimizing the electricity payment and minimizing the waiting time for the operation of each appliance in household thanks to a real-time pricing tariff combined. The output is computed with linear programming. Also in (Tsui and Chan 2012), authors study a versatile convex programming DR optimization framework for the automatic load management of various household appliances in a smart home by deploying dynamic pricing of electricity to regulate electricity consumption.
Furthermore, we can find several game-theoretic approaches concerning an energy context. One of them is the paper (Samadi et al. 2012), and its related previous work (Samadi, Schober, and Wong 2011). The model in (Samadi et al. 2012) is based on a non-cooperative game in which authors associate the users' needs with energy producers. The aim is to encourage efficient energy consumption among users with a mechanism that collects private information from users and computes the electricity bill. In this work, users receive all the required energy because the energy provider is able to buy energy from external sources.
These related works present automated approaches where users have to declare their habits and these models force in some way those habits, at the contrary, in our model, users still behave as they prefer deciding when consuming energy without communicating how they consume energy. Furthermore, we have an objective function (i.e.: available energy function) which is a parameter for payments calculation.

In our previous works (Bistarelli et al. 2016), we face an energy allocation problem modifying users' behaviour by assigning the available energy. In particular, in (Giuliodori 2016) we propose a first payment function based on Shapley value.

6 Conclusion & Future Work

Our model is able to compute accurately energy bills. It considers not only user's consumption, but also the consumption of the entire community and the disposable energy. It guarantees efficiency and fairness properties from two different perspectives: first, the community seen as a whole, then to single user point of view. Taking into account the community level, we ensure the community dummy property, community fairness, nonlinearity and ϵ-community efficiency. From the single user point of view, fairness properties are the irrelevant and parity consumer properties, virtuousness. In conclusion, our pricing scheme stimulates users a virtuous and efficient energy users' behaviour as close as possible to the availability energy function.
For what concerning future work, we can have constraints on consumption θ_i (for instance, a limit on the instant consumption). Thus, we have to study the property presented, especially the community efficiency, with these new constrained consumption functions. Furthermore, in order to prove the consistency and real applicability of our model, we have to deploy it on an agent-based simulation based on real consumption data.

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